Engineering Notes

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G80-013

Dynamics of a Flexible Body in Orbit

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V.K. Kumar* and P.M. Bainum‡
Howard University, Washington, D.C.

Nomenclature

A_n = modal amplitude
C_y = external disturbance torques
E_n = nth modal component of external disturbance forces
T = orbital period
t = time
θ = pitch angle
$\phi_\alpha^{(n)}$ = component of nth mode shape vector ($\alpha = x,y,z$)
ω_c = orbital angular velocity
ω_n = nth structural modal frequency
ξ_α = undeformed coordinates of a generic mass point ($\alpha = x,y,z$)

Introduction

FUTURE proposed space missions would involve large, inherently flexible spacecraft which are to be modeled as completely flexible bodies in orbit. Since "beam-like" structures will be the fundamental supporting element in any large scale space system, an investigation of the dynamics of a very long, slender, "beam-like" structure in orbit (Fig. 1) at very low structural frequencies is considered in this Note. The equations of motion of an arbitrary flexible body in orbit as developed by Santini[1] have been modified to a vector form[2] which may be more convenient in the analysis of structures with an arbitrary shape. In the present Note the equations of motion of a flexible beam in orbit are derived from the general equations of Ref. 2, after making the following assumptions: 1) The beam has a uniform cross-section with mass and structural properties uniformly distributed along the length. 2) The beam is straight in the undeformed state. 3) The beam is in a circular orbit. 4) The Earth's gravitational field is spherically symmetric. 5) All the motions of the beam (including elastic motions) are restricted to the orbital plane. 6) Longitudinal deformations are negligible in comparison with the lateral deflections. 7) There are no constraints for the in-plane motion of the beam.

It can be noted in Eqs. (26) and (42) of Ref. 2, [or Eqs. (15) and (17) of the conference preprint] that in the Cartesian system of coordinates the terms coupling the rigid-body rotational modes and elastic modes are linear functions of the volume integral $H_{\alpha\beta}^{(n)}$, where

$$H_{\alpha\beta}^{(n)} = \int_{\text{vol}} \xi_\alpha \phi_\beta^{(n)} \, \mathrm{d}m \qquad (\alpha,\beta = x,y,z) \tag{1}$$

Presented as Paper 78-1418 at the AIAA/AAS Astrodynamics Conference, Palo Alto, Calif., Aug. 7-9, 1978; submitted Sept. 6, 1978; revision received July 23, 1979.

Index categories: Spacecraft Dynamics and Control; Structural Dynamics; Structural Stability.

*Graduate Research Assistant, Dept. of Mechanical Engineering. Member AIAA.

†Professor of Aerospace Engineering, Dept. of Mechanical Engineering. Associate Fellow AIAA.

For very large "beam-like" structures it can be easily shown that $H_{\alpha\beta}^{(n)} = 0$ for all α and β under assumptions 5-7. This implies that the rigid-body mode is not coupled to the structural modes either through inertia or gravity. Ashley[3] also has arrived at the same conclusions regarding coupling for the case of an orbiting thin beam; however, the stability of this system at very low frequencies was not considered.[3]

Similarly, one can further note that the intermodal coupling terms which appear in the generic mode equation are linear functions of the volume integral $L_{\alpha\beta}^{(mn)}$ where

$$L_{\alpha\beta}^{(mn)} = \int_{\text{vol}} \phi_\alpha^{(m)} \phi_\beta^{(n)} \, \mathrm{d}m \qquad (\alpha,\beta = x,y,z) \tag{2}$$

For "beam-like" structures, it can be easily shown that

$$L_{\alpha\beta}^{(mn)} = \delta_{mn}\delta_{z\beta}\delta_{\alpha z} M_n$$

where δ_{mn}, $\delta_{z\beta}$, $\delta_{\alpha z}$ are Kronecker deltas, and M_n is the nth structural modal mass. Hence, it is evident that there is no coupling between the structural modes either through inertia or gravity effects. However, from examination of the coefficients of $L_{\alpha\beta}^{(mn)}$ it is seen that each structural mode is coupled to the rigid-body motion through both inertia and gravity.[2]

With the above simplifications and assumptions we arrive at the following equations for the motion of the flexible beam in orbit:

$$\ddot{\theta} + (3\omega_c^2 \sin 2\theta)/2 + C_y/2 = 0 \tag{3}$$

$$\ddot{A}_n + [\omega_n^2 - \omega_c^2(3\sin^2\theta - 1) - (\dot{\theta} - \omega_c)^2] A_n = E_n/M_n \tag{4}$$

If the rigid-body pitch oscillations are small, i.e., $\theta \ll 1$, Eqs. (3) and (4) further simplify to (assuming no external disturbances)

$$\ddot{\theta} + 3\omega_c^2 \theta = 0 \tag{5}$$

$$\ddot{A}_n + [\omega_n^2 - \dot{\theta}^2 + 2\omega_c\dot{\theta}] A_n \cong 0 \tag{6}$$

Equation (5) has the solution, $\theta = c\sin(\sqrt{3}\omega_c t + \gamma)$, where c is the amplitude of the pitch motion and γ is the phase angle. After substitution of the above solution for θ into Eq. (6), one obtains

$$\ddot{A}_n + [\omega_n^2 - 3\omega_c^2 c^2 \cos^2(\sqrt{3}\omega_c t + \gamma) + 2\sqrt{3}\omega_c^2 c\cos(\sqrt{3}\omega_c t + \gamma)] A_n = 0 \tag{7}$$

With the introduction of the dimensionless variables, $\tau = (\sqrt{3}\omega_c t + \gamma)/2$ and $z_n = A_n/l$, the generic mode equation in the following nondimensional form results:

$$\frac{\mathrm{d}^2 z_n}{\mathrm{d}\tau^2} + \frac{4}{3}\left[\frac{\omega_n^2}{\omega_c^2} - 3c^2\cos^2 2\tau + 2\sqrt{3}c\cos 2\tau\right] z_n = 0 \tag{8}$$

Using the trigonometric identity, $\cos^2 2\tau = (\cos 4\tau + 1)/2$, Eq. (8) can be rewritten as

$$\frac{\mathrm{d}^2 z_n}{\mathrm{d}\tau^2} + \frac{4}{3}\left[\left(\frac{\omega_n^2}{\omega_c^2} - \frac{3c^2}{2}\right) + 2\sqrt{3}c\cos 2\tau - \frac{3}{2}c^2\cos 4\tau\right] z_n = 0 \tag{9}$$

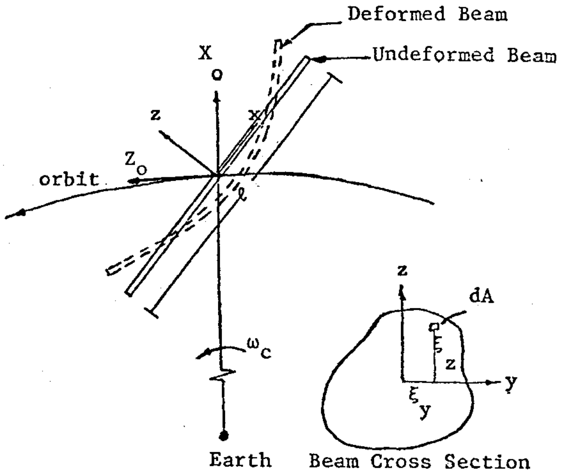

Fig. 1 Beam satellite.

Equation (9) is in the form of "Hill's 3-term equation" or "Whittaker's equation."[4] For small pitch amplitudes, i.e., $c \ll 1$, the above equation can be further approximated by the Mathieu equation:

$$\frac{d^2 z_n}{d\tau^2} + (\delta + \epsilon \cos 2\tau) z_n = 0 \tag{10}$$

where

$$\delta = 4[(\omega_n/\omega_c)^2 - 3c^2/2]/3 \qquad \epsilon = 4.62\,c$$

It can be seen from examination of the Mathieu stability diagram (Fig. 4 of Ref. 2 or Fig. 4 of the conference preprint) that for the values of δ around unity, i.e., $(\omega_n/\omega_c)^2 = O(1)$, the system may enter a region of instability. However, for large values of ω_n/ω_c the coefficient of z_n in Eq. (10) is dominated by $(\omega_n/\omega_c)^2$. Hence, in the high-frequency range, the elastic modes are essentially governed by the following simple equation:

$$\frac{d^2 z_n}{d\tau^2} + \frac{4}{3}\left(\frac{\omega_n}{\omega_c}\right)^2 z_n \cong 0 \qquad \left(\frac{\omega_n}{\omega_c} \gg 1\right) \tag{11}$$

Thus, it can be concluded that for beams with $(\omega_n/\omega_c) \gg 1$ $(n = 1,2,\ldots\infty)$, the elastic motion and the small amplitude rigid-body pitching motion are completely decoupled from each other. However, if $(\omega_n/\omega_c) = O(1)$ (highly flexible beams), one has to consider Eq. (9) to study the elastic motion, and thus the elastic motion will be coupled with the rigid-body pitching motion.

For some values of ω_n/ω_c, which are typical of highly flexible beams, numerical solutions of Eq. (9) were obtained using a digital computer, the results of which are discussed in the next section.

Simulation

It can be observed from Eq. (9) that the elastic motion of a very flexible beam is coupled with the pitching motion. For some combinations of ω_n/ω_c and pitch amplitudes, the parameters δ and ϵ in Eq. (10) may lie in the unstable region of the Mathieu stability diagram (Ref. 2). However, for larger values of ω_n/ω_c the modal response of the beam is very close to that of an harmonic oscillator. The above facts are demonstrated by the digital simulation of Eq. (9) for some typical values ω_n/ω_c as indicated. Only a few representative plots are presented in this Note.

Figure 2 shows the case where the elastic motion of the beam is unstable. The broken curve in Fig. 3 $[(\omega_n/\omega_c)^2 = 1.0, c = 0.05]$ also shows the case of an unstable

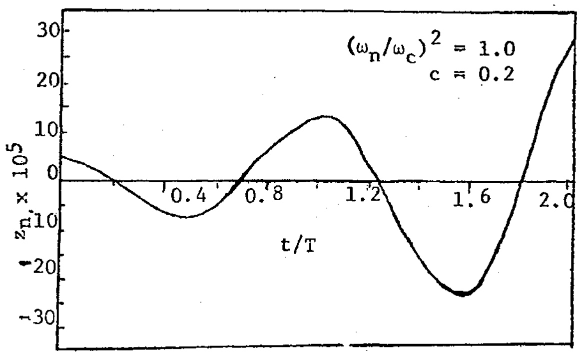

Fig. 2 Modal amplitude response—very flexible beam.

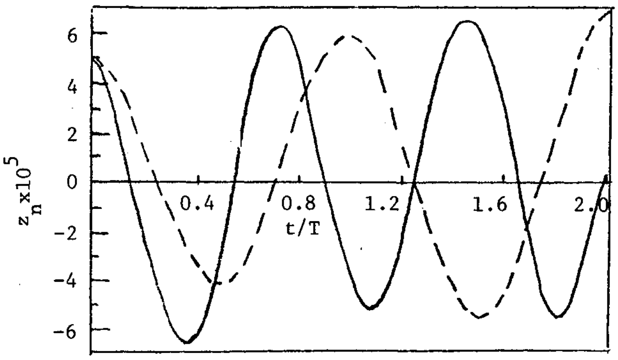

Fig. 3 Modal amplitude response—effect of increased beam stiffness and initial pitch amplitude.

elastic motion. However, in the latter case the rate of divergence is less than in Fig. 2. This may be attributed to the lower amplitude of the pitch motion. The solid curve in Fig. 3 $[(\omega_n/\omega_c)^2 = 2.0, c = 0.2]$ shows the effect of increased beam stiffness. In this case a bounded and periodic behavior of the elastic motion can be observed. We also note the significant coupling effect of pitch on the elastic motion. With a further increase in the beam stiffness the elastic motion tends to be sinusoidal and the effect of pitch on the elastic motion becomes negligible. Thus, in the higher range of natural frequencies, one can study the elastic motion of the beam independently of the rigid-body pitching motion.

Conclusions

For the case of the planar motion of a long slender beam in a circular orbit undergoing small pitch oscillations and flexural vibrations, it is seen that the pitch motion completely decouples from the elastic motion and that the elastic motion is coupled to the pitch motion and is described by a Hill's three-term equation. The possibility of parametric instability at very low natural frequencies is demonstrated in this Note. For large values of the ratio of the structural frequency to the orbital angular velocity, ω_n/ω_c, the elastic motion decouples from the pitch motion and the elastic motion closely approximates that of an harmonic oscillator.

Acknowledgments

Research supported by NASA-Grant NSG-1414. Special appreciation to R. Russell and L. Pinson of NASA Langley for their helpful suggestions.

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Thermally Induced Response of Flexible Structures: A Method for Analysis

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Harold P. Frisch*
NASA Goddard Space Flight Center, Greenbelt, Md.

Introduction

THE phenomenon of a thermally induced spacecraft instability made itself dramatically known during the flight of the OGO-IV spacecraft in the late 1960's. On that flight the 60-ft experiment boom immediately broke into a sustained large amplitude oscillation which severely compromised spacecraft performance. A detailed analysis of the problem can be found in Ref. 1. A less dramatic but still serious thermally induced instability occurred during the flight of Explorer 45 (SSS-A). On this flight, minute oscillations of the four experiment booms, caused by modulations of the thermal input, were phased in such a manner that a nutationally destablizing effect was produced. A detailed analysis of this problem can be found in Ref. 2. In addition to these postflight analyses, the work presented in Refs. 3 and 4 provides the techniques necessary for the preflight analysis of the potentiality of a thermally induced instability.

To go beyond state-of-the-art capabilities presently implies that thermally deformable appendages other than beams be considered. While this problem is of minimal importance in 1979, it will not always be so. Currently, extremely large antennas and platforms are being proposed for space application which will require shape to be controlled so as to cancel out the performance degrading effects of thermal deformation. The stability analysis of proposed shape control systems will require analysis techniques which are not currently available.

The purpose of this paper is to present a generalized approach to the problem of obtaining a computationally practical simulation model for a thermally driven structure. The formulation is such that it can be utilized in a stand-along manner or be meshed into a general multibody simulation model such as that presented in Refs. 5 and 6. As presented, the formulation is restricted to the dynamic range for which linearized radiation is a valid assumption.

Method

Assume a structure complex enough to prohibit thermoelastic continuum analysis. In this situation, finite-element techniques are inevitably used to obtain flexible-body characteristics and either finite-difference or finite-element techniques used to obtain thermal response characteristics. The problem is to dynamically couple the two disciplines so as to obtain closed-loop response information. Furthermore, the problem is compounded by the fact that the analyst requiring the multidiscipline study, say a control system analyst, usually does not have the necessary expertise to set up either a finite-element structural model or a finite-element or finite-difference thermal model. This communications problem can be partially overcome by basing the multidiscipline formulation upon currently existing, widely accessible general purpose analysis programs such as NASTRAN, SPAR, SINDA, TRASYS, DISCOS, etc.

Received July 23, 1979. This paper is declared a work of the U.S. Government and therefore is in the public domain.

Index categories: Spacecraft Dynamics and Control; Structural Dynamics; Analytical and Numerical Methods.

*Aerospace Technologist. Member AIAA.

Structural Analysis

Once the structure under investigation has been adequately defined, the analyst can usually obtain a finite-element model compatible with the preferred in-house structure's program, e.g., NASTRAN, SPAR, STARDYNE. This program will set up the following set of dynamic equations:

$$M\ddot{U}+KU=0 \tag{1}$$

where M=mass matrix, K=stiffness matrix, and U=displacement vector. Then, upon user request, the program will automatically compute a set of the most significant natural frequencies and associated vibration modes. Let N_v =total number of vibration modes computed, ω_n = nth natural frequency ($n=1,2,\ldots N_v$), and φ_n =vibration mode associated with ω_n; and normalize the modes according to the following orthonormalization condition:

$$\varphi_m^T M\varphi_n = M_T\delta_{m,n} \tag{2}$$

where M_T=scalar total mass of structure; $\delta_{m,n}$ =Kronecker Delta function, and the superscript T is used to signify "transpose of matrix."

Thermal Analysis

The most difficult portion of the study will be obtaining the required data from the thermal analyst. The underlying cause of this is the fact that general-purpose thermal analysis programs are structured to determine temperature fluctuations at particular points in the spacecraft over a temperature range which prohibits linearization of fourth-power radiation terms. For this reason, it will be best to belabor the thermal analysis portion of the study.

To start, there are two methods currently used: the finite-difference method and the finite-element method. For the purposes of this study, either approach may be taken.

If the finite-element approach is used, a set of grid points are defined which are then connected by heat-conducting elements. The difficult problem with this approach is specification of the heat-conducting elements used to simulate internal radiation effects. This gets particularly complex when the effects of shadowing must be taken into account. However, once done, the finite-element program is structured to set up the appropriate heat capacitance and conduction matrices and to numerically solve the equations.

If the finite-difference approach is used, a set of node points are defined and a general thermal balance equation is set up for each. The user is required to specify all thermal coupling between node points. While this approach at first glance appears more laborious than the finite-element approach, it provides the analyst with an added degree of flexibility which frequently is required to solve real-world thermal problems, e.g., those which must account for shadowing of internal radiation and reflection.

The "Nodal Network Thermal Balance Program," available through COSMIC[7] provides a convenient framework to build upon. Utilizing notation adapted from the program documentation, let T_I=absolute temperature at node I, T_{I_0} = average mean temperature at node I, and ΔT_I = variation from the mean at node I, such that

$$T_I = T_{I_0} + \Delta T_I \tag{3}$$